Proceedings of DETC'03
ASME 2003 Design Engineering Technical Conferences and
Computers and Information in Engineering Conference
Chicago, Illinois, September2 - 6, 2003

DETC2003/DFM-48149

MODELING VARIATION PROPAGATION THROUGH A MANUFACTURING PROCESS

Daniel Kern
Mechanical Engineering Department
Massachusetts Institute of Technology
Cambridge, Massachusetts

Anna Thornton
Analytics Inc.
Boston, Massachusetts

ABSTRACT

Companies are under increased pressure to manufacture products that have a high level of quality. Manufacturing products with a high level of quality in a cost-effective manner requires the products to be designed so that they can be manufactured with an acceptable level of variation. Creating a new design that can be produced with the necessary level of variation has historically been a very challenging problem.

A new method for calculating the effect a manufacturing process has on the mean and standard deviation of a distribution is presented. This new method is founded on the concept of characterizing a manufacturing process with two math functions called DeltaP and SigmaP. DeltaP and SigmaP represent the theory of Process Imparted Dimensional Change and Process Imparted Variation. Using these functions, closed-form solutions for the mean and standard deviation of a distribution exiting a manufacturing process can be calculated. The authors present the background of the theory as well as the derivation of the closed form solutions for the output mean and standard deviation from a generic manufacturing process. The derivation is followed by a simple example to demonstrate the method.

KEYWORDS

Variation management, Quality, Design for manufacture, Tolerance allocation, Manufacturing variation

INTRODUCTION

The goal of any manufacturing firm is to manufacture products that meet and exceed customer based requirements. Designing quality into a manufactured component is a method of accomplishing this. One aspect of quality is how closely the part conforms to the dimensions and tolerances specified on a drawing. Manufacturing Process Variation is a measure of the amount a dimension on produced parts differs from the desired target dimension.

The dimensions of produced parts can be represented statistically by a probability density function. Each probability distribution function has a mean and a standard deviation associated with it. A manufacturing process receives an input distribution and transforms it into an output distribution. Figure 1 displays a graphical representation. Manufacturing Process Variation is represented by the standard deviation of the output distribution.

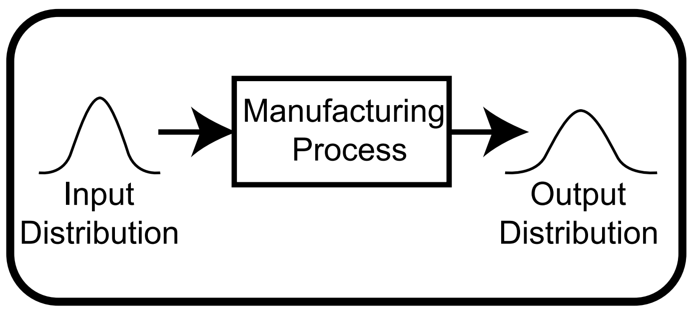

Figure 1: Process transforms a distribution

Engineers are very interested in understanding the relationship between the input and output distributions. A model of the transformation allows a design engineer to specify tolerances that can be produced with an acceptable level of quality. The same model assists a manufacturing engineer in selecting a process that ensures components are manufactured within specification. A project engineer could use the model to select suppliers based on their process capability. For example, a Design for Six Sigma [1] approach would greatly benefit from a model to calculate manufacturing output variation.

There has been a wide variety of research in the area of product and manufacturing variation. Managing the effect variation has on the final cost and quality of a product while it is being designed is explored by Thornton [2]. She developed a tool set to use during the design process to identify where variation will have significant impact.

Suri and Otto [3] use variational modeling to select manufacturing process input parameters that render the process outputs insensitive to variation. Their technique uses a designed experiment [4] to generate sensitivity functions for a manufacturing process so that its robustness can be improved.

A physics-based approach to modeling variation was introduced by Suri [5]. The approach creates an Integrated System Model that predicts the nominal values and variation of each output quality characteristic in a manufacturing system.

Frey [6] introduces the concept of a process capability matrix and bias vector to model a manufacturing system. The model developed can be used to compute the yield from the manufacturing process.

A model of how variation is transmitted through a manufacturing process is presented by Lawless, Mackay, and Robinson [7]. Their approach is to measure parts at each process in the manufacturing system and determine the amount of variation added by each process and carried through the next process. This approach is useful when the manufacturing system is in operation and measurements are easy to accurately make.

The above approaches are useful when studying different aspects of product and manufacturing variation; however, one item in variation management that has been missing is the ability to easily create useful, flexible equations for the output mean and standard deviation of a distribution exiting a manufacturing process as a function of the incoming distribution's mean and standard deviation, i.e., $\mu_{out} = f(\mu_{in},\sigma_{in})$ and $\sigma_{out} = f(\mu_{in},\sigma_{in})$.

An approach used to generate equations for the output mean and standard deviation of a distribution exiting a manufacturing process as a function of the incoming distribution's mean and standard deviation is simply to fit curves to measured data. This approach can lead to misleading results when the data used does not cover a broad operating or design range. Another downside to this approach is the inability to know what terms to include, i.e., combinations of the input mean and standard deviation and higher order terms, since the true curve is unknown. Also, the math functions have no real meaning or significance to an engineer, thus using them can be misleading and testing their validity is difficult.

A new method of modeling the transformation a distribution goes through when acted upon by a manufacturing process is presented in this paper. This new method is simple and quick to use. The method results in closed-form equations for the output mean and output standard deviation. The method also produces mathematical graphs for a process that can be used to quickly compare two processes qualitatively.

PROCESS IMPARTED DIMENSIONAL CHANGE

Manufacturing processes are the building blocks of a production line. A manufacturing process transforms the dimensions of a component. The process acts on an incoming dimension and attempts to transform it to a desired target value.

> **Desired Dimensional Change**, M, is the change a process must make to a part's dimension to result in the dimension being equal to the desired (target) value.

The desired dimensional change will be referred to as M. An equation for M can be written as

$$M = \mu_{target} - x \quad (1)$$

where μ_{target} is the desired target value for the output dimension and x is a component's incoming dimension.

A process is setup to produce a distribution with a desired target mean, μ_{target}; however, the output distribution does not always have a mean equal to the target mean. The difference is often referred to as the mean shift or the bias, b. The equation for bias is seen below in Equation 2.

$$b = \mu_{output} - \mu_{target} \quad (2)$$

A process transforms an input dimension into an output dimension, e.g., a lathe reduces an incoming diameter to a smaller diameter. Process Imparted Dimensional Change is the actual change a dimension undergoes when operated on by a process when there is no process variation present. For example, an incoming distribution of rod diameters has a mean of 25mm. The output target mean for a turning process is 20mm. A particular rod with a diameter of 26mm is operated on and results in a rod with a diameter of 20.1mm. In this case, the Process Imparted Dimensional Change is -5.9mm for a desired dimensional change, M, of -6.0mm.

> **Process Imparted Dimensional Change** is the actual change a dimension undergoes when operated on by a process when there is no process variation present.

Process Imparted Dimensional Change is different than bias. The bias is the actual difference between the target mean and the mean of the output distribution. Process Imparted Dimensional Change is the change a single dimension undergoes when a process operates on the part in the absence of variation.

Process Imparted Dimensional Change is a function of process parameters. Typical parameters to consider are the desired dimensional change, work piece material, cutter speed, feed, type of machine, and type of tool, etc. Most other parameters that affect Process Imparted Dimensional Change are specific to the process being modeled.

A special case of Process Imparted Dimensional Change is when all process parameters except M, desired dimensional change, are fixed. This special case is named DeltaP.

DeltaP is a mathematical representation of the actual change a dimension undergoes when operated on by a process with a given set of parameters when no process variation is present. It is a function of the desired dimensional change, M.

$$DeltaP = f(M) \qquad 3$$

A method of generating the function DeltaP is discussed later. A sample graph of DeltaP versus desired dimensional change is shown in Figure 2.

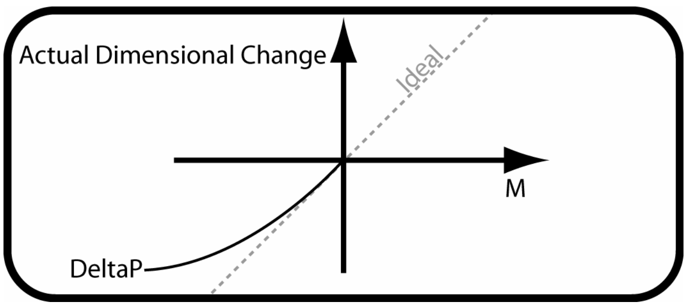

Figure 2: Graph of sample DeltaP

As seen in Figure 2, the ideal manufacturing process would have an associated Delta P = M. In this case, a manufacturing process produces all dimensions exactly equal to the output target dimension.

A DeltaP exists for a manufacturing process with a fixed set of operating parameters. Therefore, a particular process can have an infinite number of DeltaPs, one for each set of operating parameters. For example, a lathe has one DeltaP when it is operating on aluminum at 1500 RPM, and the same lathe operation has another DeltaP when operating at 2000 RPM.

PROCESS IMPARTED VARIATION

Process Imparted Variation is the variation an under control process inherently imparts on an incoming distribution that has zero variance. For example, a lathe is set to machine rods to a desired output diameter. A hundred rods with identical diameters are machined by this process. The result is a distribution of rod diameters with a mean and a standard deviation. This standard deviation is the Process Imparted Variation.

Process Imparted Variation is the standard deviation an under control process inherently imparts on an incoming distribution that has zero variance.

Similar to Process Imparted Dimensional Change, Process Imparted Variation is a function of process parameters. Typical parameters to consider are amount of material removed, work piece material, cutter speed, feed, type of machine, and type of tool. Most other parameters that affect Process Induced Variation are specific to the process.

A special case of Process Imparted Variation is when all process parameters except M, desired dimensional change, are fixed. This special case is named SigmaP.

SigmaP is a mathematical representation of the standard deviation an under control process with a set of given parameters inherently imparts on an incoming distribution that has zero variance. It is a function of the desired dimensional change, M.

$$SigmaP = f(M) \qquad 4$$

The exact method of generating the function SigmaP is discussed in a later section. A sample graph of SigmaP versus Desired Dimensional Change is shown in Figure 3.

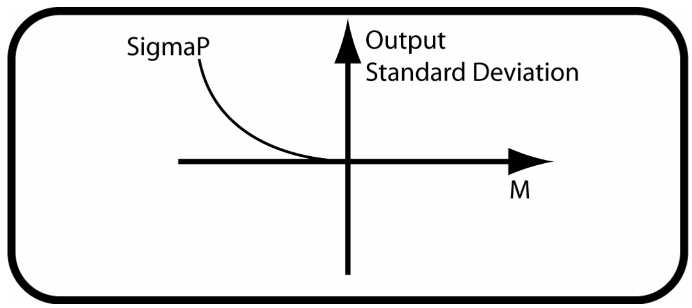

Figure 3: Graph of sample SigmaP

QUALITATIVE USE OF DeltaP AND SigmaP

The theory of a manufacturing process being represented by the two mathematical functions of DeltaP and SigmaP can be used as a powerful qualitative tool.

The DeltaPs for a process at different operating parameters or for different processes can be reviewed to select a process that more accurately produces dimensions at the desired value. For example, a process with a DeltaP that is almost a 45° line would result in dimensions closer to the desired value than a process with a DeltaP having a smaller slope. DeltaP can also be used to evaluate how sensitive a process is to the incoming dimension. For example, a highly non-linear DeltaP may reveal the need to set the process up to remove less material each pass to ensure output dimensions close to the target value.

The SigmaPs can also be used to compare how much variation different processes impart to a distribution. For example, a process with a low, flat SigmaP would be preferred to one with a steep slope. A SigmaP curve can also reveal a process' "sweet spot" where the variation imparted could be minimized.

DERIVATION OF QUANTITATIVE USE OF DeltaP AND SigmaP

The qualitative uses of Delta P and SigmaP can be beneficial to an engineer; however, the authors' main goal is to develop equations to calculate the mean and variance of a process' output distribution given the input mean and variance. This goal requires the following derivation.

Binning of distributions

The first step in deriving the equations for the output mean and output variance is to bin the incoming distribution. Binning is the procedure of dividing a distribution into discrete divisions, or bins. Once the width of a bin is fixed, the height is set so that the area of the bin is equal to that under the probability distribution function over that range. Therefore, the areas in all the bins add to unity. The area in a bin can be calculated regardless of the type of probability density function according to Equation 5. A picture of a sample binned input distribution is shown in Figure 4.

$$Area_i = \int_{x_{i-1}}^{x_i} \left(pdf_{input} \right) dx \qquad 5$$

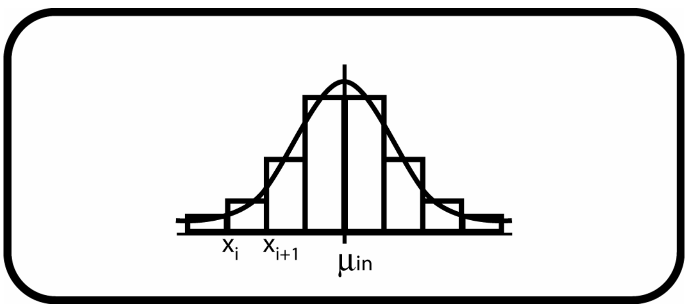

Figure 4: Sample of a binned input distribution

The bin width and the number of bins, N, are arbitrary at this point. The smaller the bin width, the greater number of bins required to traverse the original distribution. Also, as the bin width decreases, the discrete representation approaches that of the original distribution.

Processing the bins

The next step is to treat each bin as an individual distribution. Each bin will be "processed" using the theory of DeltaP and SigmaP, resulting in an output sub-distribution.

As an approximation, each bin is considered to contain all equal dimensions. For example, the ith bin from x_{i-1} to x_i contains components that all have the same dimension, equal to $\frac{\left(x_{i-1}+x_i\right)}{2}$. (This approximation becomes mute at a later point.) Using this approximation, the Desired Dimensional Change for the ith bin is calculated to be

$$M_i = \mu_{target} - \frac{\left(x_{i-1}+x_i\right)}{2} \qquad 6$$

Each bin is processed to create an output sub-distribution as shown in Figure 5. Using the definitions and equations for DeltaP and SigmaP, the mean, μ_i, and standard deviation, σ_i of the output sub-distribution can be approximated as

$$\mu_i \cong \frac{x_{i-i}+x_i}{2} + DeltaP\left(M_i\right) \qquad 7$$

$$\sigma_i \cong SigmaP\left(M_i\right) \qquad 8$$

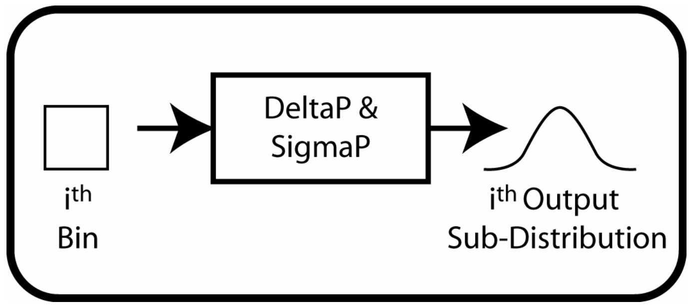

Figure 5: Bin transformed into output sub-distribution

After every bin has been "processed," the result is N sub-distributions. Each sub-distribution has a mean and standard deviation that has been calculated using Equations 7 and 8.

Combining the sub-distributions

The main goal is still to derive equations to calculate the mean and standard deviation of a manufacturing process' output distribution. At this point, the means and standard deviations are known for the N distributions created by "processing" the bins of the input distribution. The manufacturing process output distribution is simply the combination of these N sub-distributions. The distribution that results from mixing multiple distributions into one is often referred to as a mixture distribution. The equations for calculating the mean and variance of a mixture distribution are shown below.

$$\mu_{mixed} = p_1\mu_1 + p_2\mu_2 + \ldots + p_N\mu_N \qquad 9$$

$$\begin{aligned}\sigma^2_{mixed} &= \left(p_1\sigma_1^2 + p_2\sigma_2^2 + \ldots + p_N\sigma_N^2\right) \\ &+ \left(p_1\mu_1^2 + p_2\mu_2^2 + \ldots + p_N\mu_N^2\right) \\ &- (p_1\mu_1 + p_2\mu_2 + \ldots + p_N\mu_N)^2\end{aligned} \qquad 10$$

The μ_i's are the means of the distributions being mixed, and the σ_i's are the standard deviations of the distributions being mixed. The p_i term is equal to the number of measurements in the i^{th} distribution divided by the total number of measurements.

Equations 9 and 10 can be used to generate equations for the approximate values of an output distribution's mean and standard deviation. The p_i terms are replaced with Equation 5 and Equations 7 and 8 replace the means and standard deviations. These substitutions result in the following equations.

$$\mu_{output} \cong \sum_{i=1}^{N} \left\{ \int_{x_{i-1}}^{x_i} \left(pdf_{input} \right) dx * \left[\frac{x_{i-1} + x_i}{2} + DeltaP\left(M_i \right) \right] \right\} \quad \textbf{11}$$

$$\begin{aligned} \sigma^2_{output} \cong & \sum_{i=1}^{N} \left(\int_{x_{i-1}}^{x_i} \left(pdf_{input} \right) dx * \left[SigmaP\left(M_i \right) \right]^2 \right) \\ & + \sum_{i=1}^{N} \left(\int_{x_{i-1}}^{x_i} \left(pdf_{input} \right) dx * \left[\frac{x_{i-1} + x_i}{2} + DeltaP\left(M_i \right) \right]^2 \right) \\ & - (\mu_{output})^2 \end{aligned} \quad \textbf{12}$$

Equations 11 and 12 are approximations because they were derived from a finite number of bins because of the bin approximation made earlier.

Manufacturing processes often generate dimensions that are normally distributed [8]. With this assumption, the pdf_{input} terms in Equations 11 and 12 can be replaced with the equation for a normal distribution. At this time, it is also advantageous to standardize the equations above to use the reduced variable z in place of x. This allows for the standard form of the normal distribution, mean of zero and one standard deviation is at z equal to 1, equation to be used. The main advantage to this substitution is the limits on the integrals can be generalized.

The equation for a normal distribution is

$$pdf_{Normal} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{ -\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right\} \quad \textbf{13}$$

The reduced variable z is related to x according to

$$z = \frac{(x - \mu)}{\sigma} \quad \textbf{14}$$

The standard normal distribution is

$$pdf = \frac{1}{\sqrt{2\pi}} \exp\left\{ -\frac{z^2}{2} \right\} \quad \textbf{15}$$

Equations 6 and 14 are combined to result in the equation below for the Desired Dimensional Change as a function of the reduced variable z.

$$M_i = \mu_{t\arg et} - \left\{ \frac{\left\{ z_{i-1} + z_i \right\} * \sigma_{input}}{2} + \mu_{input} \right\} \quad \textbf{16}$$

Now the appropriate substitutions can be made into Equations 11 and 12. Equation 15 is substituted in for the pdf_{input} terms. Equation 16 replaces the M_i terms. Finally, the x's are translated into z's using Equation 14. The results of these changes are the following equations.

$$\mu_{output} \cong \sum_{i=1}^{N} \left\{ \begin{array}{l} \dfrac{1}{\sqrt{2\pi}} * \displaystyle\int_{z_{i-1}}^{z_i} \left\{ \exp\left(-\frac{z^2}{2} \right) \right\} dz \\ * \left[\begin{array}{l} \dfrac{\left\{ z_i + z_{i+1} \right\} * \sigma_{input}}{2} + \mu_{input} \\ + DeltaP\left(\mu_{t\arg et} - \left\{ \dfrac{\left\{ z_{i-1} + z_i \right\} * \sigma_{input}}{2} + \mu_{input} \right\} \right) \end{array} \right] \end{array} \right\} \quad \textbf{17}$$

$$\begin{aligned} \sigma^2_{output} \cong & \sum_{i=1}^{N} \left(\begin{array}{l} \dfrac{1}{\sqrt{2\pi}} * \displaystyle\int_{z_{i-1}}^{z_i} \left(\exp\left\{ -\frac{z^2}{2} \right\} \right) dz * \\ \left[SigmaP\left(\mu_{t\arg et} - \left\{ \dfrac{\left\{ z_{i-1} + z_i \right\} * \sigma_{input}}{2} + \mu_{input} \right\} \right) \right]^2 \end{array} \right) \\ & + \sum_{i=1}^{N} \left(\begin{array}{l} \dfrac{1}{\sqrt{2\pi}} * \displaystyle\int_{z_{i-1}}^{z_i} \left(\exp\left\{ -\frac{z^2}{2} \right\} \right) dz * \\ \left[\begin{array}{l} \dfrac{\left(z_{i-1} + z_i \right) * \sigma_{input}}{2} + \mu_{input} \\ + DeltaP\left(\mu_{t\arg et} - \left\{ \dfrac{\left\{ z_{i-1} + z_i \right\} * \sigma_{input}}{2} + \mu_{input} \right\} \right) \end{array} \right]^2 \end{array} \right) \\ & - (\mu_{output})^2 \end{aligned} \quad \textbf{18}$$

Making approximate solutions exact

Equations 17 and 18 are one step closer to the goal; however, they are still only approximate solutions because of the original assumption of a finite number of bins. The solutions are made exact by taking the limit as the bin width goes to zero.

Several changes occur when the bin width goes to zero. This limit converts a discrete function into a continuous one. The discrete variable z_i becomes the continuous variable z. The integral over the normal distribution becomes the value of the integrand at that value of z. The summations become integrals with limits of negative infinity to positive infinity so that it spans the entire distribution. Making these modifications and simplifying results in the following.

$$\begin{aligned}\mu_{output} &= \mu_{input} \\ &+ \int_{-\infty}^{+\infty} \left\{ \begin{array}{l} \frac{1}{\sqrt{2\pi}} * \left\{ \exp\left(-\frac{z^2}{2} \right) \right\} \\ * \left[DeltaP\left(\mu_{target} - \left\{ z * \sigma_{input} + \mu_{input} \right\} \right) \right] \end{array} \right\} dz \end{aligned} \qquad \mathbf{19}$$

$$\begin{aligned}\sigma^2_{output} &= \int_{-\infty}^{+\infty} \left(\begin{array}{l} \frac{1}{\sqrt{2\pi}} * \left(\exp\left\{ -\frac{z^2}{2} \right\} \right) * \\ \left[SigmaP\left(\mu_{target} - \left\{ z * \sigma_{input} + \mu_{input} \right\} \right) \right]^2 \end{array} \right) dz + \sigma^2_{input} \\ &+ \mu^2_{input} + \int_{-\infty}^{+\infty} \left(\begin{array}{l} \frac{1}{\sqrt{2\pi}} * \left(\exp\left\{ -\frac{z^2}{2} \right\} \right) * \\ \left[\begin{array}{l} 2z\sigma_{input} * \\ DeltaP\left(\mu_{target} - \left\{ z * \sigma_{input} + \mu_{input} \right\} \right) \end{array} \right] \end{array} \right) dz \\ &+ 2\mu_{input} * \left\{ \mu_{output} - \mu_{input} \right\} \\ &+ \int_{-\infty}^{+\infty} \left(\begin{array}{l} \frac{1}{\sqrt{2\pi}} * \left(\exp\left\{ -\frac{z^2}{2} \right\} \right) * \\ \left[\left[DeltaP\left(\mu_{target} - \left\{ z * \sigma_{input} + \mu_{input} \right\} \right) \right]^2 \right] \end{array} \right) dz \\ &- (\mu_{output})^2 \end{aligned} \qquad \mathbf{20}$$

The above equations are exact solutions for the mean and variance of an output distribution. The equations require knowledge of DeltaP and SigmaP to be of practical use. This is the topic of the next section.

Forms of DeltaP and SigmaP

Until this point in the derivation, the functions DeltaP and SigmaP have remained in the equations for μ_{output} and σ_{output}. General forms of DeltaP and Sigma P are presented below in Equations 21 and 22.

$$\begin{aligned} DeltaP(M) &= a_0 + a_1 M + a_2 M^2 \\ &+ \ldots + a_i M^i + \ldots + a_\alpha M^\alpha \end{aligned} \qquad \mathbf{21}$$

$$\begin{aligned} SigmaP(M) &= b_0 + b_1 M + b_2 M^2 \\ &+ \ldots + b_k M^k + \ldots + b_\beta M^\beta \end{aligned} \qquad \mathbf{22}$$

In this paper for simplicity, the functions DeltaP and SigmaP will be approximated as quadratic equations. This approximation matches closely with simulation results and reduces the chance of "over fitting" the curves. The simplified forms of DeltaP and SigmaP are below.

$$DeltaP(M) = a_0 + a_1 M + a_2 M^2 \qquad \mathbf{23}$$

$$SigmaP(M) = b_0 + b_1 M + b_2 M^2 \qquad \mathbf{24}$$

Closed-form solution for μ_{output} and σ_{output}

Finally, closed form equations for μ_{output} and σ_{output} can be generated. Equations 23 and 24 are substituted into Equations 19 and 20, expanded, and then simplified to produce the following.

$$\begin{aligned} \mu_{output} &= \mu_{input} + a_0 + a_1 \mu_{target} \\ &- a_1 \mu_{input} + a_2 \mu^2_{input} \\ &- 2a_2 \mu_{input} \mu_{target} \\ &+ a_2 \mu^2_{target} + a_2 \sigma^2_{input} \end{aligned} \qquad \mathbf{25}$$

$$\begin{aligned} \sigma^2_{output} &= b_0^2 + b_1^2 \left\{ \mu^2_{input} + \sigma^2_{input} - 2\mu_{input}\mu_{target} + \mu^2_{target} \right\} \\ &- 2b_1 b_2 \left[\mu_{input} - \mu_{target} \right] \\ &* \left[\mu^2_{input} + 3\sigma^2_{input} - 2\mu_{input}\mu_{target} + \mu^2_{target} \right] \\ &+ 2b_0 \left\{ \begin{array}{l} b_1 \left[\mu_{target} - \mu_{input} \right] \\ + b_2 \left[\mu^2_{input} + \sigma^2_{input} - 2\mu_{input}\mu_{target} + \mu^2_{target} \right] \end{array} \right\} \\ &+ b_2^2 \left[\begin{array}{l} \mu^4_{input} + 3\sigma^4_{input} - 4\mu^3_{input}\mu_{target} \\ + 6\sigma^2_{input}\mu^2_{target} + \mu^4_{target} \\ + 6\mu^2_{input} \left\{ \sigma^2_{input} + \mu^2_{target} \right\} \\ - 4\mu_{input} \left\{ 3\sigma^2_{input}\mu_{target} + \mu^3_{target} \right\} \end{array} \right] \\ &+ \sigma^2_{input} + \mu^2_{input} + 2\mu_{input} * \left\{ \mu_{output} - \mu_{input} \right\} \\ &- (\mu_{output})^2 + a_0^2 + \\ &a_1^2 \left\{ \mu^2_{input} + \sigma^2_{input} - 2\mu_{input}\mu_{target} + \mu^2_{target} \right\} \\ &+ 2a_0 \left\{ \begin{array}{l} a_1 \left[\mu_{target} - \mu_{input} \right] \\ + a_2 \left[\mu^2_{input} + \sigma^2_{input} - 2\mu_{input}\mu_{target} + \mu^2_{target} \right] \end{array} \right\} \\ &- 2a_1 \left\{ \begin{array}{l} \sigma^2_{input} + a_2 \left[\mu_{input} - \mu_{target} \right] \\ * \left[\mu^2_{input} + 3\sigma^2_{input} - 2\mu_{input}\mu_{target} + \mu^2_{target} \right] \end{array} \right\} \\ &+ 4a_2 \sigma^2_{input} \left[\mu_{input} - \mu_{target} \right] \\ &+ a_2^2 \left[\begin{array}{l} \mu^4_{input} + 3\sigma^4_{input} - 4\mu^3_{input}\mu_{target} \\ + 6\sigma^2_{input}\mu^2_{target} + \mu^4_{target} \\ + 6\mu^2_{input} \left\{ \sigma^2_{input} + \mu^2_{target} \right\} \\ - 4\mu_{input} \left\{ 3\sigma^2_{input}\mu_{target} + \mu^3_{target} \right\} \end{array} \right] \end{aligned} \qquad \mathbf{26}$$

Although Equation 26 appears to be complicated, it is merely a function of μ_{input}, σ_{input}, μ_{target}, DeltaP coefficients, and SigmaP coefficients. The above equations can easily be input into a spreadsheet or other computer program to compute the mean and standard deviation of a process' output distribution for different values of input distributions' means and standard deviations if DeltaP and SigmaP are known for the given process.

CALCULATING DeltaP AND SigmaP

DeltaP and SigmaP are constructed using manufacturing data related to the process of interest. Using the second order polynomial forms of DeltaP and SigmaP allows Equations 25 and 26 to be used to find coefficients by fitting manufacturing data to them.

The first step is to locate values for μ_{input}, σ_{input}, μ_{target}, μ_{output}, and σ_{output} for the process of interest. Next, guesses are made for the coefficients. The guesses are used with the sample μ_{input}, σ_{input}, and μ_{target} to calculate values for μ_{output} and σ_{output}. Non-negative error terms are computed by subtracting these calculated values from the actual values of the sample data and squaring the result. The coefficients are then adjusted, and the squared error is recalculated. Repeating this until the sum of the squared errors is minimized identifies the best set of coefficients. These steps are easily implemented using the solver feature in a spreadsheet application, employing a dedicated math application, or writing a stand-alone computer program.

EXAMPLE

A very simple example is presented to demonstrate one possible use of the method presented in this paper. The example is conveyed as a fictional story about an engineer making decisions concerning the appropriate raw stock and equipment to use to produce a new component.

Johnny is an engineer for ABC Products. He is responsible for designing and manufacturing a new component that has a circular cross section. The most important dimension of the rod is its diameter. Based on functional requirements, the rod's diameter is specified to be 20.0 mm with a lower limit of 18.5 mm and an upper limit of 21.5 mm. Johnny has decided that he will be happy if the manufactured rod diameters have a C_{pk} of at least 3. The equation for C_{pk} is shown below.

$$c_{pk} = Min\left(\frac{\left(\mu_{output} - LL\right)}{3\sigma}, \frac{\left(UL - \mu_{output}\right)}{3\sigma}\right) \qquad 27$$

Johnny can choose from three different types of circular raw bar stock. The raw stock types all have the same material properties, but they vary in diameter. The cost is another variable. The cost is higher for raw stock that has less variation. The stock distribution parameters are given in Table 1 below.

Table 1: Raw circular stock parameters

	Mean [mm]	Standard Deviation [mm]	Cost per Piece [$]
Stock A	23.0	0.30	0.70
Stock B	24.0	0.12	1.20
Stock C	25.4	0.70	0.60

Johnny also has three lathes at his disposal. He has sample manufacturing data for each of the lathes. The data is presented below in Table 2.

Table 2: Sample manufacturing data

	Input Mean [mm]	Input Standard Deviation [mm]	Output Mean [mm]	Output Standard Deviation [mm]
Lathe 1	27.005	0.451	21.139	0.149
	24.993	0.305	20.629	0.080
	21.999	0.070	20.120	0.011
	25.002	0.103	20.628	0.049
	24.997	0.100	20.625	0.047
Lathe 2	26.516	0.330	22.608	0.659
	24.801	0.121	21.524	0.367
	23.410	1.017	20.868	0.504
	22.704	0.597	20.553	0.258
	21.001	0.049	20.080	0.021
Lathe 3	27.036	0.582	20.097	0.057
	24.679	0.453	20.059	0.036
	24.003	0.118	20.048	0.026
	23.204	0.201	20.036	0.021
	21.000	0.049	20.005	0.003

Johnny places the data of Table 2 into a spreadsheet and uses Equation 25 with a solver to solve for the coefficients of DeltaP. He then repeats the process with the same data, the calculated coefficients of DeltaP, and Equation 26 to calculate the coefficients of SigmaP. The results are shown below in Table 3.

Table 3: Calculated DeltaP and SigmaP coefficients

		Process 1	Process 2	Process 3
DeltaP	a_0	-0.050	-0.072	-0.008
	a_1	0.949	0.901	0.987
	a_2	0.017	0.048	0.000
SigmaP	b_0	-0.010	-0.082	-0.014
	b_1	-0.007	-0.056	-0.011
	b_2	0.001	0.008	0.000

The values above allow Johnny to plot the DeltaP and SigmaP functions for the three Lathes. The plots of DeltaP appear below in Figure 6. The plots of SigmaP appear below in Figure 7. The plots are for the DeltaP and SigmaP functions for each lathe with a given set of particular operating parameters. The lathes were fixed with these operating parameters when they produced the data shown in Table 2.

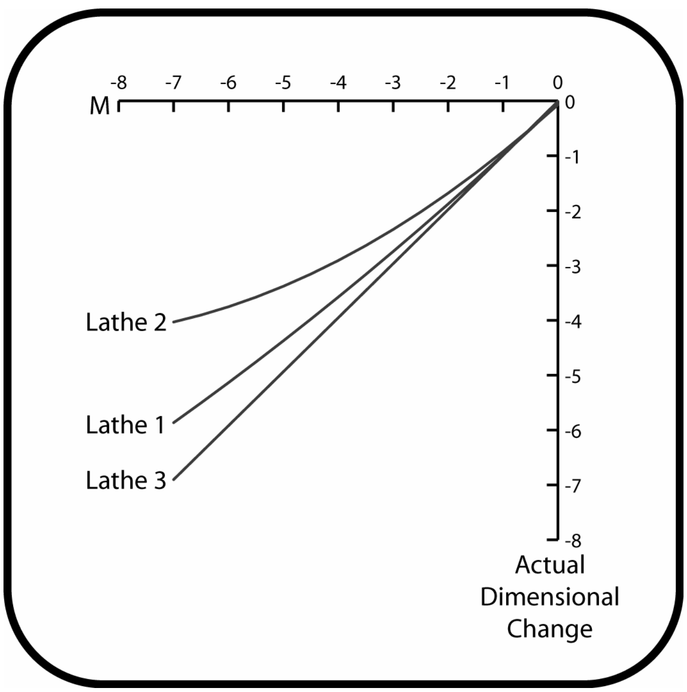

Figure 6: Graph of DeltaP functions

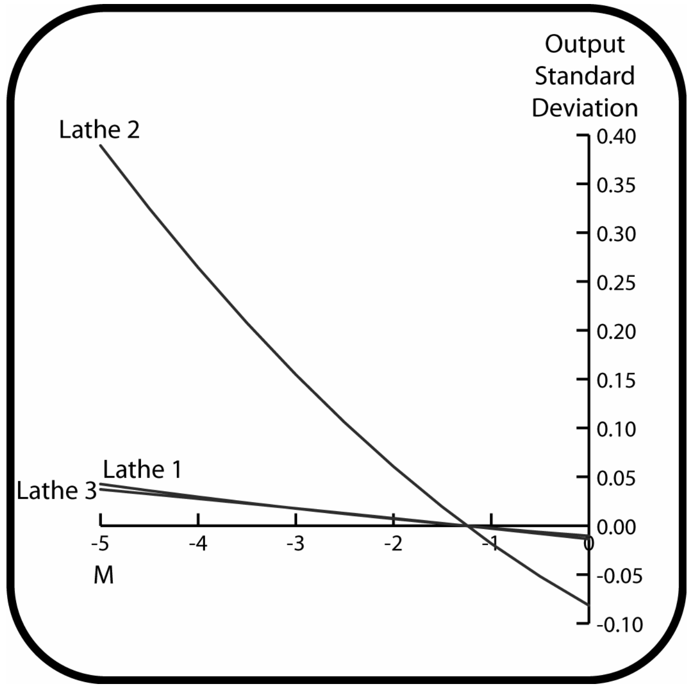

Figure 7: Graph of SigmaP functions

Based on the graphs above, Johnny quickly discerns that Lathe 2 is going to produce parts of inferior quality compared to Lathe 1 and Lathe 3. He comes to this conclusion because the Lathe 2 DeltaP is the furthest away from the ideal 45° line and the Lathe 2 SigmaP has the steepest slope of the three lathes. He also determines Lathe 3 will be more likely to produce parts close to the target mean, even if he has to use oversized incoming stock, since its DeltaP almost has a 45° slope. Remember, ideally, the actual dimensional change is equal to the desired dimensional change, which results in a line with a 45° slope. He is unable to decide between Lathe 1 and Lathe 3 based solely on the graphs since their curves are so close to each other, and he would like to consider the total cost of each part; therefore, quantitative analysis is required.

Johnny knows the cost to machine one diameter with each lathe. The costs are summarized in Table 4 below.

Table 4: Machining costs

	Machining Cost per Piece [$]
Lathe 1	0.75
Lathe 2	0.70
Lathe 3	1.00

It is easy to see there are nine possible combinations of stock selection and process selection to produce the desired component. Using Equation 25, Equation 26, Equation 27, the data in Table 1, the data in Table 3, and the data in Table 4, Johnny populates Table 5 with a C_{pk} value and a cost for each of the nine combinations.

Table 5: Possible process and stock combinations

Lathe	Stock	C_{pk}	Cost [$]
2	C	n/a	1.30
2	B	0.502	1.90
2	A	1.424	1.40
1	C	1.508	1.35
1	**A**	**8.436**	**1.45**
1	B	9.687	1.95
3	C	11.127	1.60
3	B	17.484	2.20
3	A	26.415	1.70

Based on the values in Table 5 and remembering his minimum C_{pk} requirement of 3, Johnny decides to use Stock A and Lathe 1. This combination is the lowest cost alternative that generates a Cpk greater than 3.

CONCLUSION

Managing the variation of a dimension while a component is being designed is a powerful ability. The authors presented a new method for calculating the effect a generic manufacturing process has on an incoming distribution's mean and standard deviation. This method allows an engineer to calculate the mean and standard deviation an output distribution has as it exits a manufacturing process. The foundation of this method is the theory of Process Imparted Dimensional Change and Process Imparted Variation that was introduced and used in the form of DeltaP and SigmaP

Future work in this area of research will extend into the area of system level modeling. The math models representing manufacturing processes will be linked together to study the effect a series of processes has on a distribution. These models will also be combined with math models representing assembly operations. This addition will allow engineers to calculate the variation of dimensions of an assembly.

REFERENCES

[1] Antony, J. and R. Coronado, "Design for six sigma." *Manufacturing Engineer*. 2002. p. 24 - 26.

[2] Thornton, A.C., "Variation risk management using modeling and simulation." Journal of Mechanical Design, 1999. **121**(2): p. 297-304.

[3] Suri, R. and K. Otto. "System-level robustness through integrated modeling." *ASME International Conference on Design Theory and Methodology*. 1999. Las Vegas, Nevada.

[4] Phadke, M., *Quality Engineering Using Robust Design*. 1989, Englewood Cliffs, New Jersey: Prentice Hall. 334.

[5] Suri, R., "An integrated system model for variation reduction in manufacturing systems." *Mechanical Engineering*. 1999, MIT: Cambridge, MA. p. 159.

[6] Frey, D., "Using product tolerances to drive manufacturing system design." *Mechanical Engineering*. 1997, MIT: Cambridge, MA. p. 365.

[7] Lawless, J.F., R.J. MacKay, and J.A. Robinson, "Analysis of variation transmission in manufacturing processes." Journal of Quality Technology, 1999. **31**(2): p. 131 -154.

[8] Kalpakjian, S. and S. Schmid, *Manufacturing Engineering and Technology*. Fourth Edition ed. 2001.